one would immediately ask whether this formulation may be used for similar large deflection problems such as buckling. The answer is that although for a certain class of problems this may indeed be advantageous, the general applicability is limited by the fact that if boundary conditions are imposed on the radial displacement

$$v(\ell)=\zeta(\ell)-\frac{1}{2}\int_0^{\ell}\left[\frac{\partial\xi}{\partial y}\right]^2 dy \qquad (3)$$

then the boundary condition becomes nonlinear although the differential equations remain linear. This point also underscores the general necessity of considering simultaneously the flexural and axial degrees of freedom in large deflection problems. It should also be pointed out that the Vigneron formulation trades the advantage of kinematic orthogonality ($\dot\xi$ and $\dot v$ are orthogonal velocity componets, $\dot\xi$ and $\dot\zeta$ are not) for structural simplicity. Conceivably there may be problems for which such trades are not worthwhile.

This commentator is in agreement with Professor Likins that the "partial linearization" method will remain one of the most useful methods. Unfortunately the radial beam example given in Ref. 1 represents a somewhat incorrect application of the method. Dr. Vigneron's formulation is certainly a viable alternative approach which deserves further exploration.

References

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[2] Vigneron, F. R., "Comment on Mathematical Modeling of Spinning Elastic Bodies for Modal Analysis," *AIAA Journal,* Vol. 13, Jan. 1975, pp. 126-127.

Reply to Bertrand T. Fang

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DR. FANG correctly points out that the method outlined in Ref. 2 does not depend on the suppression of the extensional displacement, ζ. The suppression is a convenient approximation, which may be introduced to shorten the derivation somewhat.

Dr. Fang further notes that the boundary condition on $v(\ell)$ is nonlinear (Eq. (3) of his Comment). This does not necessarily preclude the possibility of use of the formulation for buckling problems. As an example, one may consider the case where Ω and $\partial(\;)/\partial t$ are equal to zero, bending is in one plane only ($\eta=0$), and a constant axial load, P, is applied at $y=\ell$. The work done by the load (negative potential) is the proportional to $-Pv(\ell)$, or

$$-P\{\zeta(\ell)-\tfrac{1}{2}\int_0^{\ell}\left(\frac{\partial\xi}{\partial y}\right)^2 dy-\zeta(0)\}$$

Application of the principle of virtual work, with the above and the correspondingly simplified potential of Eq. (6), Ref. 2, leads to

$$EA\,\zeta_y=0$$

$$EI\,\xi_{yyyy}-P\,\xi_{yy}=0,$$

together with appropriate boundary conditions. The latter equation may be recognized as that associated with the study of buckling of beam columns. More general extensions of the formulation along these lines are given in Ref. 2.

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Comment on "On the Issue of Resonance in an Unsteady Supersonic Cascade"

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VERDON and McCune[1] have reported that the iterative procedures used in their linear unsteady supersonic cascade analysis failed to converge for certain combinations of the cascade parameters. The range in which divergence occurred was given as

$$|\sigma+kMx_A|\leq k(x_A^2-\mu^2y_A^2)^{1/2} \qquad (1)$$

where $k=\omega M\mu^{-2}$, $\mu^2=M^2-1$, M is the freestream Mach number, x_A and y_A are the cascade stagger and normal gap distances, respectively, and ω and σ are the frequency and interblade phase angle of the blade motion. (Note that only an interblade phase angle variation of 2π is of physical interest and hence σ can be restricted to the range $-\pi<\sigma\leq\pi$.) Since the publication of Ref. 1, numerical procedures have been developed by the present author which provide results within the foregoing range, but not at its end points

$$\sigma+kMx_A=\pm k(x_A^2-\mu^2y_A^2)^{1/2} \qquad (2)$$

At these points the numerical approach used for evaluating the sum of an infinite series kernel function, given by Eq. (24) of Ref. 1, fails. On this basis it was suspected that this infinite series might be divergent for such parametric combinations, indicating resonant operation; however, this conjecture could not be proved. Thus, the proof appearing in Ref. 2 on the divergence of the kernel function for parameter values satisfying Eq. (2) is a most welcome contribution. This work is particularly useful since it generalizes the earlier work of Samoilovich,[3] which was only recently brought to this author's attention by Kurosaka. Further, Samoilovich's result appears to have been achieved by a formal mathematical demonstration rather than by a rigorous proof.

Resonance appears to be the logical consequence of linear theories. The conditions given by Eq. (2), or more generally by Eq. (10) of Ref. 2, are formally identical to those obtained for a subsonic cascade. In addition, resonance has the same physical interpretations for both subsonic and supersonic flows, including the result that the blades cannot support un-

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